

Complex Algebraic Geometry

Oral Exam Syllabus

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Examiners: Ron Donagi, Tony Pantev

Time: Apr 30, 10:00AM

References

[L] John Lee, *Introduction to Complex Manifolds*, American Mathematical Society, 2024.

[H] Daniel Huybrechts, *Complex Geometry: An Introduction*, Springer, 2005.

[GH] Phillip Griffiths, Joseph Harris, *Principles of Algebraic Geometry*, Wiley, 1978.

Examination Material

Fundamentals: [H], Ch 2

- Divisors, line bundles, Picard groups.
- Projective spaces.
- Blowups.
- Canonical divisors and adjunction formula.
- Almost complex structures, holomorphic and antiholomorphic tangent space, Dolbeault operators and Dolbeault cohomology.

Cohomology: [L], Ch 5-6

- Sheaves and their cohomology.
- Long exact cohomology sequence.
- Acyclic resolutions

Connections: [H] Ch 4

- Connections
- Curvatures.
- Chern classes.

Kähler Manifolds: [L] Ch 8

- Hermitian and Kähler metrics.

Hodge Theory: [L] Ch 9

- Hodge theory on Kähler manifolds.
- Lefschetz theorems.
- Applications of Hodge theory: Poincaré duality, Serre duality, Hodge-Dolbeault theorem, Riemann-Roch theorem.

Kodaira Theorems: [L] Ch 10

- Kodaira vanishing theorem.
- Kodaira embedding theorem.

Curves: [GH] Ch 2.1-2.3

- Riemann-Hurwitz theorem.
- Abel-Jacobi theory.
- Riemann-Roch for curves.
- Canonical embedding and hyperelliptic curves.
- Curves of low genus.